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# APPLICATIONS OF NON-LINEAR VISCOELASTICITY AND CUMULATIVE DAMAGE
## (A Realistic Evaluation of Real Propellant Behavior)

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Prepared For

Department of the Navy
Naval Ordnance Systems Command (ORD-0331)
Contract No. N00017-70-C-4441

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# I. INTRODUCTION AND SUMMARY

## A. OBJECTIVES

With the development of new propellant materials, like the hydroxyl-terminated polybutadiene propellants (HTPB),[1] grain designs with inner-bore hoop strains in excess of 15% can be employed; while successful performances at strains up to 40% have been observed. At these strain levels the nonlinear response effects of propellants are highly significant, even exceeding the broad statistical limits bounding the linear engineering theories. In anticipation of the requirements to be imposed by these newly developing propellants, we plan to evaluate their nonlinear viscoelastic behaviors and reduce to practice the associated analytical methods. These studies will center upon the theories recently developed by Farris[2,3] at the University of Utah.

A strong recommendation for the study of nonlinear properties of solid propellants was made by the ICRPG Working Group on Mechanical Behavior, at its Nov. 1968 meeting.[4]

> "2. Development of Stress-strain Relations for Nonlinear Materials. This includes classical analytic methods for nonlinear viscoelastic materials with complex geometries and should also involve studies of thermomechanical coupling as well as the development of new numerical methods capable of handling thermomechanical effects and methods of three dimensional analysis. Research in the area should extend to experimental and microstructural methods where studies should be continued and increased so that the study of path and history dependent behavior of both linear and nonlinear materials can be evaluated and predicted. These developments should include improved characterization methods for nonlinear viscoelastic materials."

Our current studies meet the objectives proposed by this working group.

The most advanced and most pertinent failure criterion for solid propellants is the linear cumulative damage criterion (coupled with the maximum principal stress criterion) used by Bills.[5-9] This relation requires, however, a precise definition of the pertinent grain stresses as a function of time. For grains seeing only small strains and simple histories, the existing linear viscoelastic stress analysis is adequate. But, for the larger inner-bore strains, or for more complex histories under small strains, nonlinear viscoelastic analyses will probably be required for accurate failure predictions.

Pending the development of these analyses an empirical approach to failure predictions is recommended. This approach involves a strain failure criterion in which appropriately designed test specimens are subjected to the same strain-time-temperature history that the grain is expected to experience. A small effort demonstrating this approach is planned.

In summary, the primary objective of the current effort is to provide a realistic evaluation of real propellant behaviors, with emphasis upon the effects of larger strains and their attendant marked nonlinear viscoelastic behavior. To accomplish this primary objective, we shall: (1) evaluate the nonlinear viscoelastic response properties of solid propellants in terms of the theories of Farris[2,3], and reduce to practice the associated analytical methods; and (2) develop a practical, empirical method of calculating cumulative damage in terms of a strain failure criterion.

## B. SUMMARY

The proper design of solid propellant rockets requires that meaningful stress analyses for the propellant grain be available to permit calculation

of the state of stress or strain in the propellant structure at all times. To accomplish this objective, mechanical equations of state governing the stress-strain behavior of the rate, temperature, and history sensitive solid propellant must be known. Existing constitutive theories are not capable of predicting the response for one-dimensional, small strain, isothermal deformation of solid propellant, including the effect of previous strain history. It is planned to investigate and apply a recently developed constitutive theory which appears to describe real propellants quite well. The theory incorporates a non-fading memory feature into the constitutive equations that is lacking in the equations used to date.

The new constitutive theory, which was developed by Farris at the Universtiy of Utah, accounts for the stress-softening which propellants exhibit after deformation. The stress-softening, or the "Mullins' Effect" as it is commonly called, is due to the breakdown of the propellant microstructure with deformation. Farris' theory attributes much of the time dependent behavior (such as relaxation and creep) to the time-dependence of the Mullins' effect, and provides, for the first time, a constitutive law which provides an adequate description of real behavior.

Propellants will be characterized according to this new constitutive theory to demonstrate its validity in controlled isothermal and thermo-viscoelastic laboratory experiments. In accomplishing this objective, it will be necessary to: (1) develop material characterization methods; (2) modify existing analytical computational programs; and (3) perform thermo-viscoelastic model tests.

In considering the practical problems of damage prediction, we have reexamined the use of strain criteria of failure, since the strains in a motor can be readily measured, unlike the stresses, and are not substantially time dependent. Thus, the maximum strain on identical repeat temperature cycles will show essentially the same value whereas the maximum stress has been shown (on occasion) to increase, and to be presently incalculable with desired precision. It is planned, therefore, to develop a practical method of calculating cumulative damage based on a strain failure criterion, and demonstrate its applicability in a limited number of laboratory tests. Correlations between laboratory-based failure predictions and actual motor failures will be made using available data.

The status of the nonlinear analyses are discussed next.

## II. A NEW THEORY FOR NONLINEAR VISCOELASTICITY

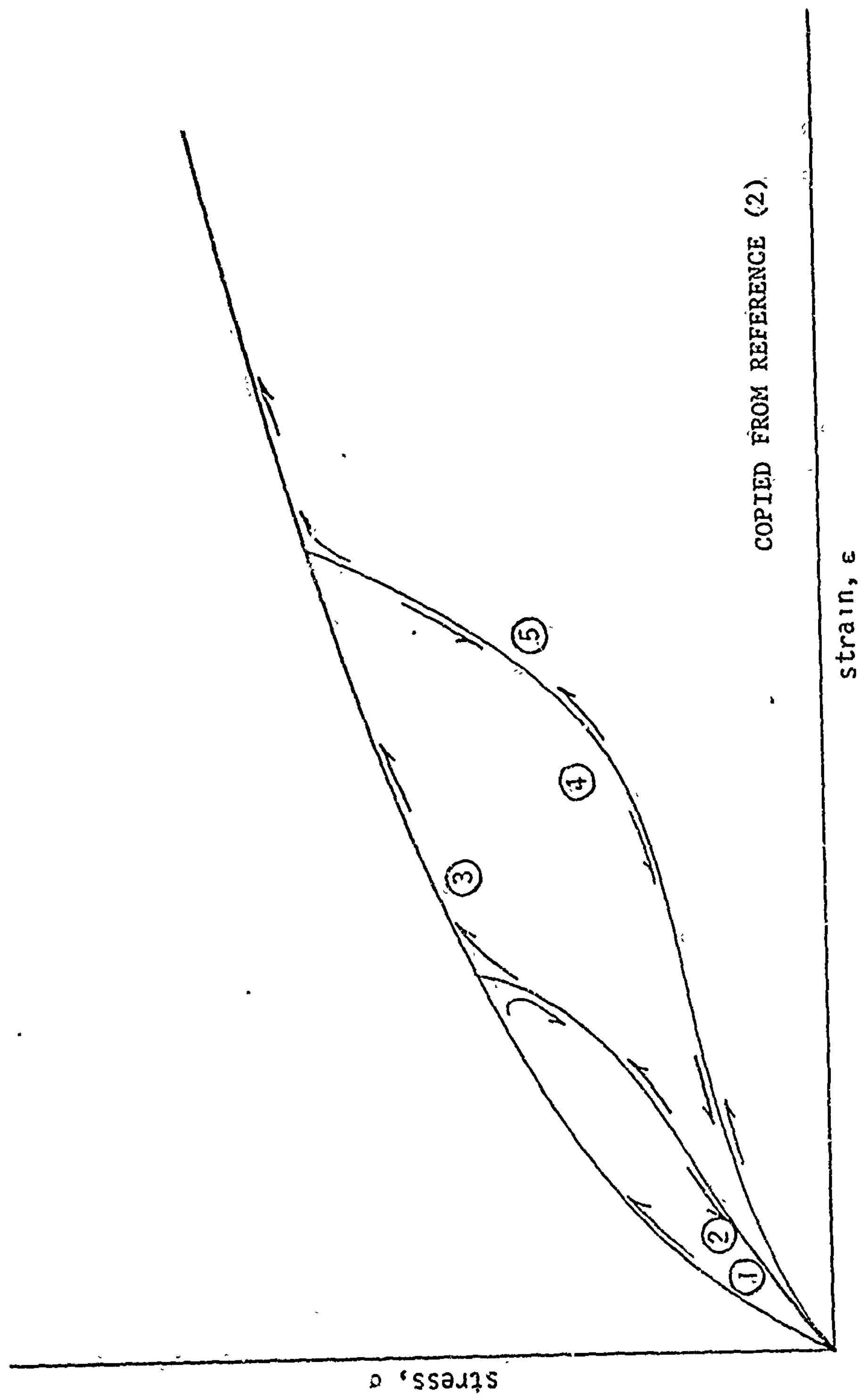
This subject is approached by discussing in the first section the "Mullins' Effect" and its relation to "memory" in the propellant. Since a propellant remembers its past deformation history, it does not meet the classical definition of linear viscoelastic material. Considerations of the memory of past deformations leads to a "non-fading memory" constitutive theory, which is discussed in the second section. The analytical capability at Aerojet Solid Propulsion Company to incorporate the nonlinear viscoelastic relations into existing thermoviscoelastic stress analyses is described in the third section. The fourth section summarizes the current efforts.

## A. THE MULLINS' EFFECT IN SOLID PROPELLANTS

Viscoelastic materials have a "memory"; that is, their present state depends upon their entire past history. Nevertheless, all of the viscoelastic constitutive theories used to date for polymeric materials, such as solid propellants, are based on the concept of "fading memory". This means that a material is more sensitive to its immediate past than to its distant past. Since a propellant remembers its past deformation history, it does not meet the classical definition of linear viscoelastic material, instead a "non-fading" memory constitutive theory must be used. Experience indicates that propellants do not fall into the category of fading memory materials even at small strains below detectable dewetting or volume dilatation.

There is considerable evidence that all the hysteresis effects observed in propellants and much of the viscoelastic behavior are caused by the time dependent failure of the polymer on a molecular basis. At near equilibrium rates and small strains, propellants exhibit the same type of hysteresis that many lightly filled, highly crosslinked rubbers demonstrate at large strains.[10,11,12] This phenomenon is called the "Mullins' Effect" and has been attributed to microstructural failure. Mullins postulated that a breakdown of particle-particle association and possibly also particle-polymer breakdown could account for the effect. Later Bueche proposed a molecular basis for the "Mullins' Effect".

Figure 1 illustrates a typical hysteresis behavior for repetitive stretching of a propellant to increasing strain levels. In highly crosslinked rubbers, the effect only depends upon strain and is irreversible.[10] In




A schematic set of stress-strain curves illustrating the Mullins Effect in lowly filled polymers at large strains.



Figure 1

propellants, however, there is considerable evidence that the "Mullins' Effect" is a very strong function of time.

B. NEW CONSTITUTIVE LAW

The new constitutive law for non-fading memory materials was originally developed from a microstructural model assuming that the gradient of strain at the microscopic level in propellants was caused by the filler particles.[13] The model also assumes that the polymer in the high strain region fails according to a linear cumulative damage criterion. The non-linear theory was developed in a manner similar to that for linear viscoelasticity; the nonlinear material obeying the rule of scalar linearity but failing on the rule of superposition. As mentioned above, nearly all propellants fall into this category. We are looking, then for a mathematical equation that satisfies the following operator equations

$$\sigma\{C\varepsilon(t)\} = C\sigma\{\varepsilon(t)\}, \quad \text{All scalars } C \tag{1}$$

$$\sigma\{\varepsilon_1(t) + \varepsilon_2(t)\} \neq \sigma\{\varepsilon_1(t)\} + \sigma\{\varepsilon_2(t)\} \tag{2}$$

The first of these equations is called a homogeneous equation of degree one and there are many non-linear functions of more than one variable that satisfy it.

Continuum mechanics tells us that the stress can be at most some functional of the strain history.

$$\sigma(t) = \underset{s=0}{\overset{t}{F}} \{\varepsilon(t-s)\} \tag{3}$$

For an elastic material, the functional reduces to a function. Now there is only one variable of the present value of strain that will result in a rate independent material that is homogeneous to degree one. That variable is the strain itself, $\varepsilon$. Hence, just as in linear elasticity the strain is a variable in our constitutive law. If we now look for homogeneous functionals of the history of degree one, we find permissible variables are

$$||\varepsilon||_p = \left\{\int_0^t |\varepsilon(\xi)|^p d\xi\right\}^{1/p}, \quad 1 \leq p \leq \infty \quad . \tag{4}$$

where $\xi$ is a dummy time variable

This variable, or set of variables $||\varepsilon||_p$, $1 < p < \infty$ are called norms in the mathematical field of real analysis. One finds that $||\varepsilon||_p$ can be constant and is simply the maximum value of the strain in the history of the deformation while all other $||\varepsilon||_p$, $p < \infty$ are time functions. Hence, for the stress-strain law to be time independent, the only permissible variables are $\varepsilon$ and $||\varepsilon||_\infty$, the maximum value of $||\varepsilon||_p$ occurring over the history of the test. We find the most general constitutive law for a material with complete recovery satisfying our homogeneity condition becomes

$$\sigma = \varepsilon G \left[(\varepsilon/||\varepsilon||_\infty)^2\right] \tag{5}$$

where G is a function of the variable $(\varepsilon/||\varepsilon||_\infty)^2$ and

$||\varepsilon||_\infty$ is a scalar measure of the strains seen in the past.

The reason the variable must be squared is that we must allow the operator equation to hold for negative and positive scalars.

Now on the first stretch, the available value of $||\varepsilon||_\infty$ must equal $\varepsilon$, whence Equation (5) becomes

$$\sigma = \varepsilon G(1) \tag{6}$$

where G(1) is the first stretch modulus function.

After the first pull, subsequent loading will show a hysteresis path with $||\varepsilon||_\infty$ still equal to the highest value of $\varepsilon$ on the first pull ($\varepsilon = \varepsilon_o$) and the stress will be described by Equation (5) which becomes

$$\sigma = \varepsilon G(\varepsilon/\varepsilon_o)^2 \tag{7}$$

This behavior is illustrated in the sketches given in Figure 2.

Similarly, a one-dimensional law that is time dependent for a material exhibiting complete recovery and is totally of the non-fading memory type can be written as

$$\sigma = \varepsilon \sum_p G_p \left[ (\varepsilon / ||\varepsilon||_p)^2 \right] \tag{8}$$

As an example, consider a case where the value of the function G gives Equation (8) the form

$$\sigma = 100\ \varepsilon \left[ 1 + (\varepsilon / ||\varepsilon||_p)^{2n} \right] \tag{9}$$

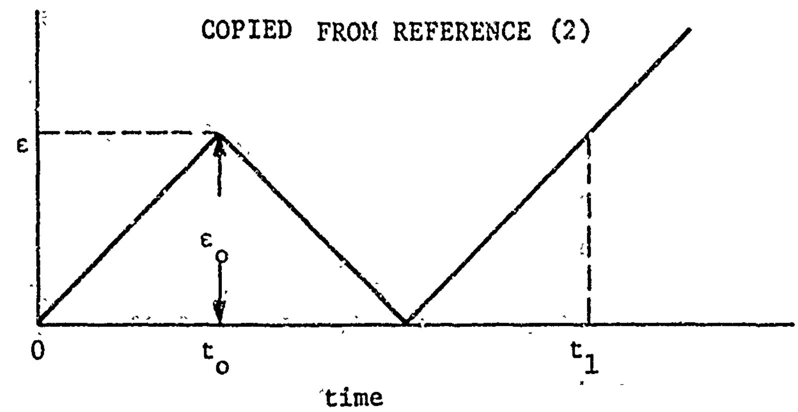
The output for a jump relaxation test as given by this equation can be shown to be

$$\frac{\sigma}{\varepsilon_o} = 100 \left[ 1 + t^{-2n/p} \right] \tag{10}$$

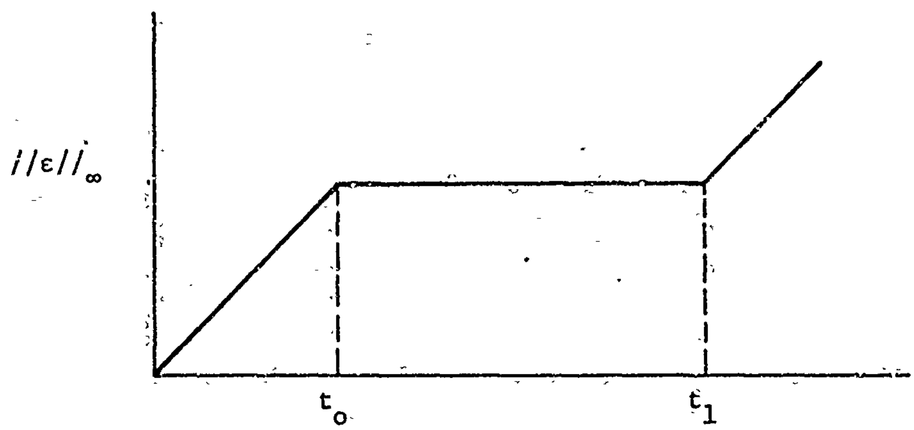
The response to this simplified version of our non-fading memory equation and a linear viscoelastic material having the same relaxation modulus are given in Figures 2 through 6 for many typical inputs. Examinations of these figures indicates that very simple equations can describe the peculiar behavior of many solid propellants whereas the linear viscoelastic equations do not.

STRESS-STRAIN BEHAVIOR ACCORDING TO EQUATION (5)

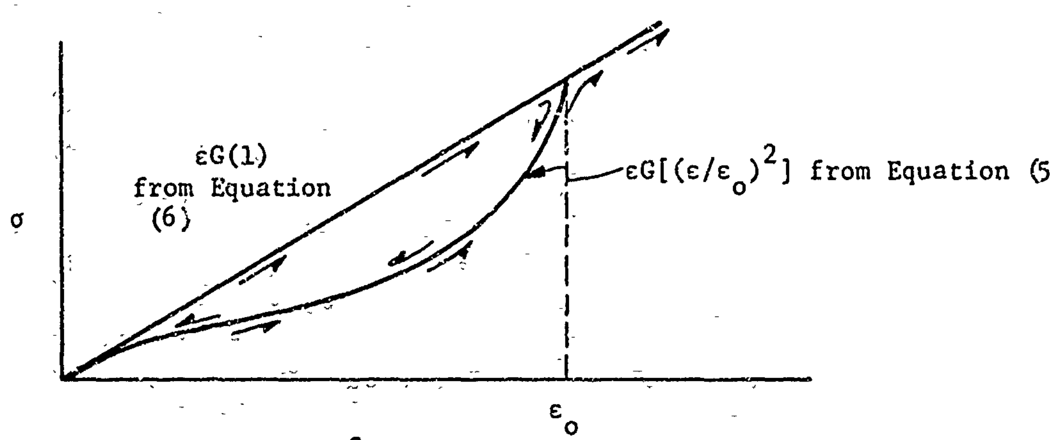
COPIED FROM REFERENCE (2)




a. The strain history will be

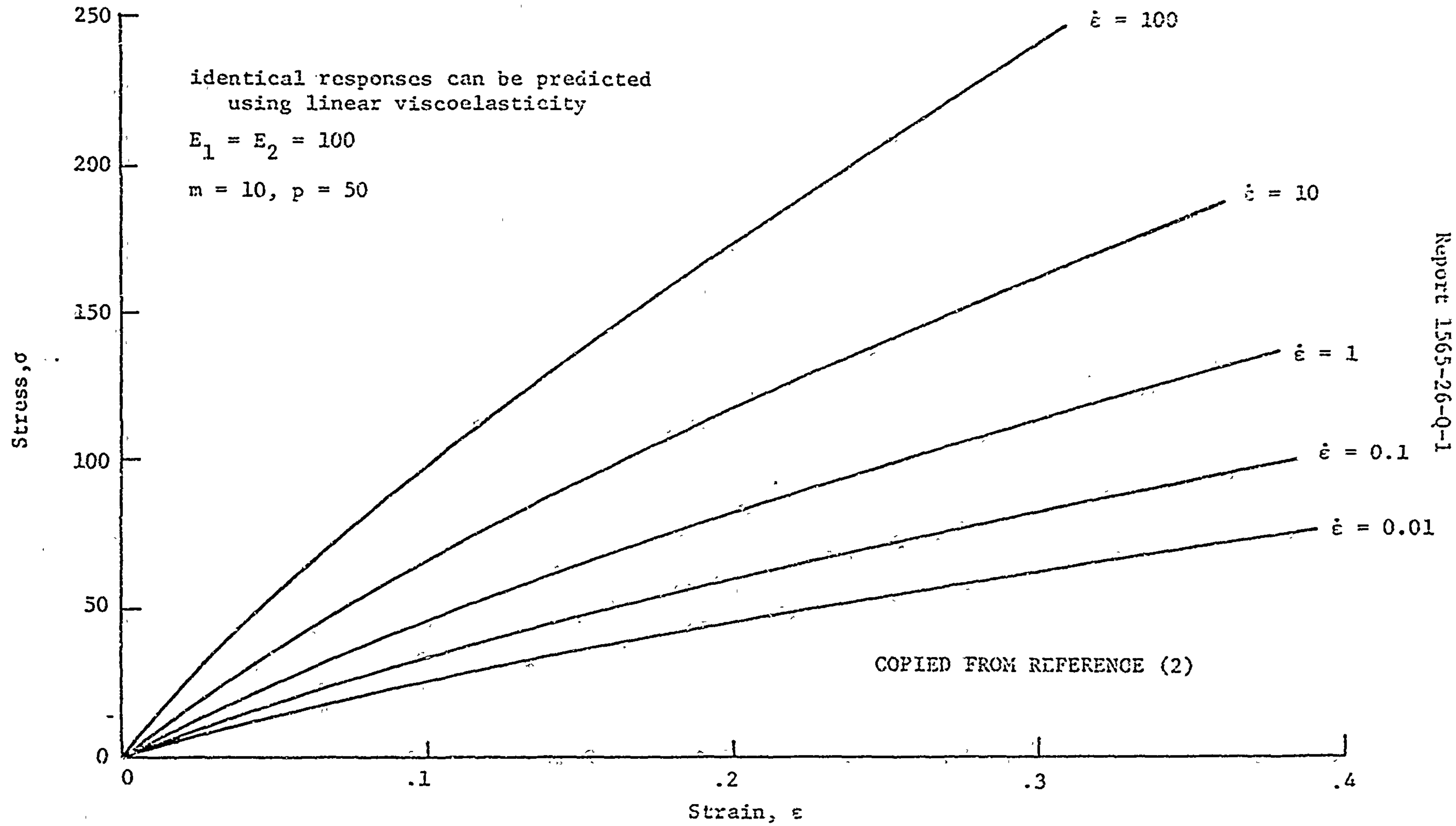



b. The variable $//\varepsilon//_\infty$ will be



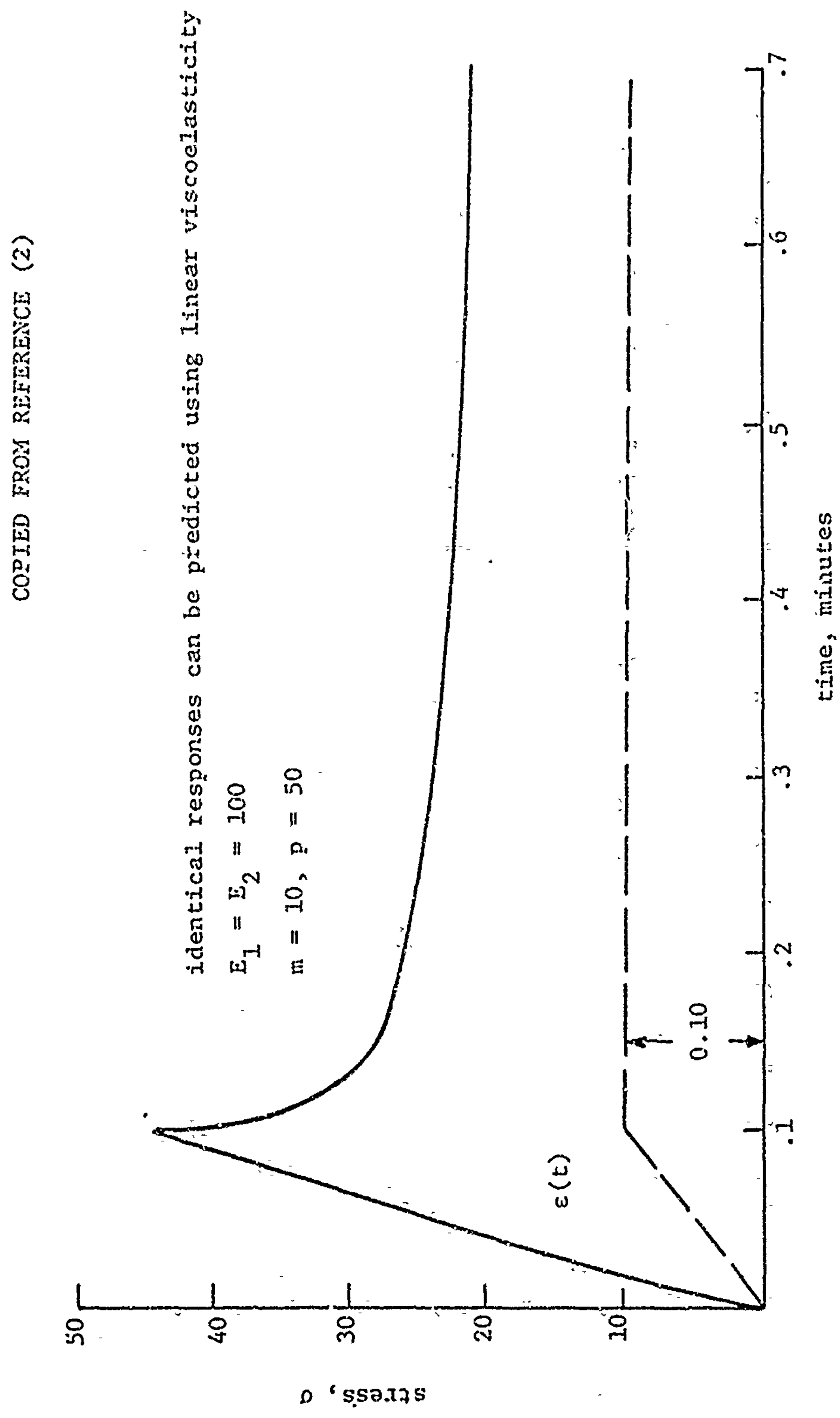

c. and the stress-strain behavior will be

Figure 2




CALCULATED CONSTANT RATE STRESS-STRAIN BEHAVIOR USING EQUATION 9

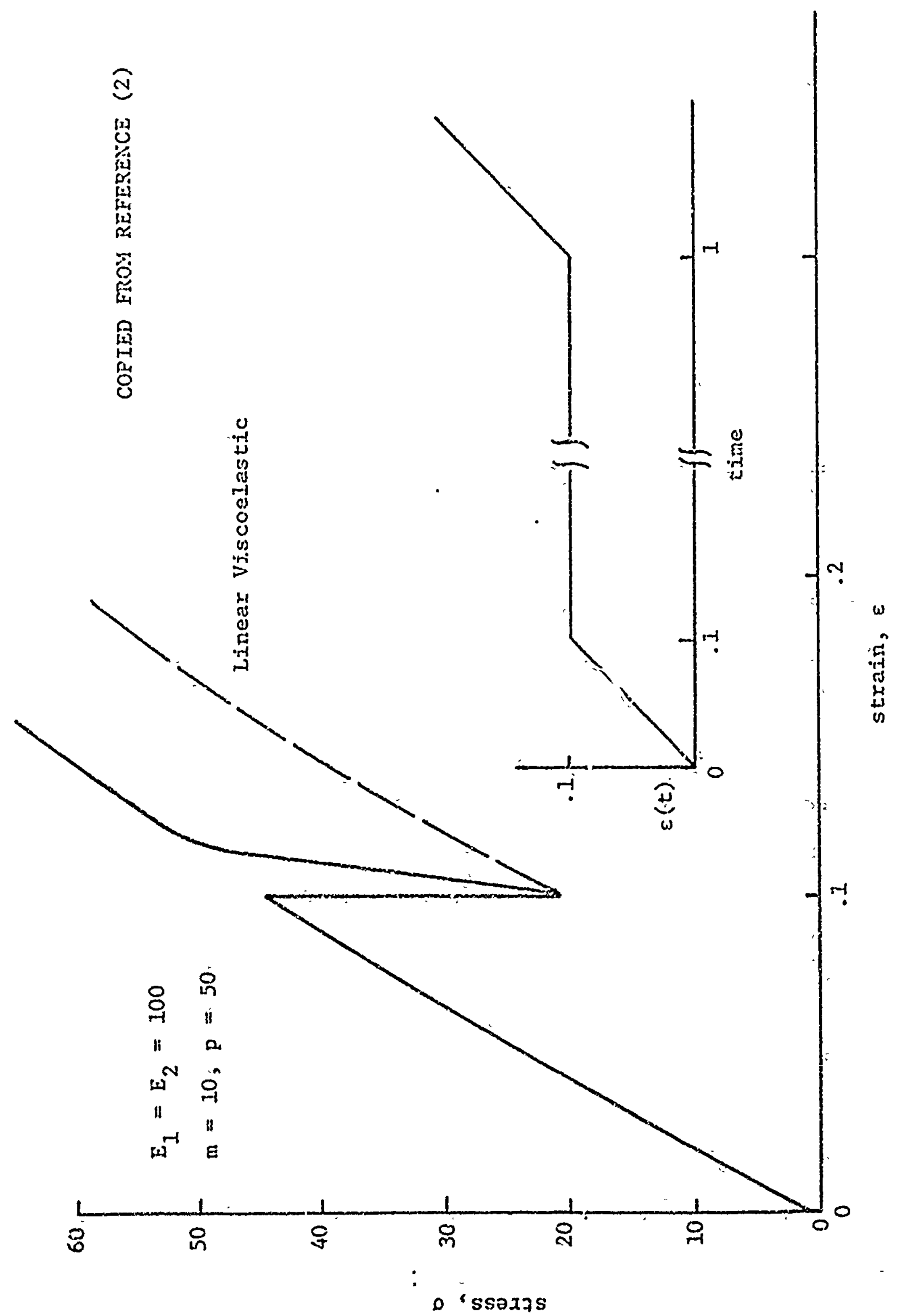
Figure 3




CALCULATED RAMP STRAIN STRESS RELAXATION BEHAVIOR USING EQUATION 9



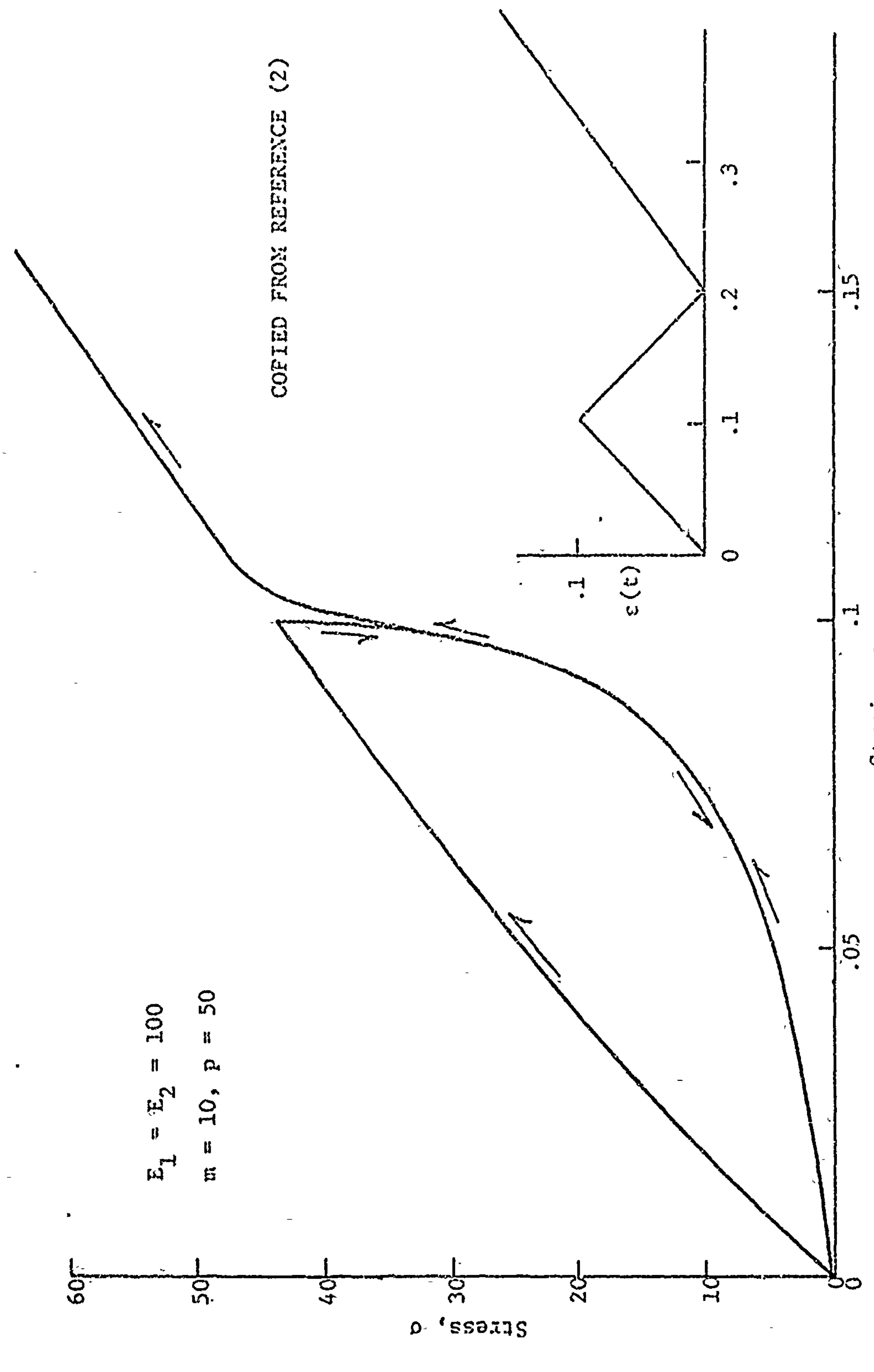
Figure 4




CALCULATED STRESS-STRAIN RESPONSE TO AN INTERRUPTED RAMP INPUT USING EQUATION 9



Figure 5




CALCULATED HYSTERESIS RESPONSE TO A REVERSED RAMP STRAIN INPUT USING EQUATION

Figure 6

About the only forms of analyses that have been used to date on three dimensional problems are the linear elastic and the linear viscoelastic analyses. Nonlinear constitutive theories make nearly impossible the solution to the resulting nonlinear system of equations even for simple geometries. Interestingly enough, our constitutive equation, although not linear in the general sense, does not require nonlinear solution procedures for certain classes of engineering problems. Some forms of three dimensional stress analysis are readily handled by such a non-fading memory constitutive law. The particular case of proportional loading encompasses a very large class of engineering problems and can readily be solved with our constitutive law using linear elastic solutions for the displacements. For such a loading we find that any constitutive theory which only satisfies the first linearity rule will admit a linear elastic solution for the internal displacements or strains. The stresses can then be calculated from the history of the strains using the constitutive law. This holds true for linear elasticity, linear viscoelasticity, as well as any other law of homogeneity of degree one such as ours.

The question arises for a three dimensional analysis, as to what additional variables can be admitted for our non-fading memory law, since there are now six stresses and six strains. In the one dimensional equation we used $(\varepsilon/||\varepsilon||_p)^2$, $1 < p < \infty$ as our new variable. Now in an elastic solution for proportional loading, when we double the surface displacements we double the internal displacements, and hence the internal strains (all six of them). For this case we find that all homogeneous measures of the state of strain at a point, such as any of the strain invariants, all reduce to the same basic variable. Since only homogeneous measures of strain can be used, we find an analogy between all our constitutive laws.

## C. ANALYTICAL BACKGROUND

The capability of the industry in developing numerical procedures for the structural analysis of propellant grains is based upon extensive experience. The stress function formulation and finite difference solution developed by Messner and Shearly[14] made possible the elasticity solution of case bonded motors of arbitrary axisymmetric geometry for the first time. This procedure was then combined with approximate inversion techniques of LaPlace transform to obtain time dependent solutions for a large class of practical propellant grain problems. Development work by Herrmann while at Aerojet[15] provided the key to applying finite element techniques to incormpressible materials. This formulation was included in Brisbane and Becker's work[16] which has become the standard of the industry for routine propellant grain analyses. More recently, Herrmann and Peterson developed a numerical procedure for viscoelastic stress analyses.[17] This procedure utilizes the conventional constitutive law for a linear, thermo-rheologically simple viscoelastic material. The procedure incorporating this law has been programmed to handle transient thermal problems in case bonded cylindrical grains. Results of these transient thermal solutions have been used in conjunction with the linear cumulative damage theory in earlier studies in this area.[5-8]

It is planned to extend this procedure to include the newly developed constitutive law. This should overcome the problems observed on a number of programs where the measured grain stresses vary by a factor of up to six from those predicted. (For example, the Structural Test Vehicle Program results)[18]

The most pertinent application of the new results will be to predict thermal and mechanical stresses in simplified models which permit experimental verification of the analyses.

D. CURRENT EFFORTS

It is planned to investigate and apply the Farris constitutive theory, which appears to describe real propellant behavior quite well. This will entail three main efforts, first, the empirical characterization of propellants by the new relation, second, incorporation of the pertinent form of the constitutive relation into a one-dimensional, thermoviscoelastic stress analysis, and third, a demonstration of the resulting engineering analysis with tests on model propellant structures. These are discussed below.

1. Characterization Studies

Before any experimental work can begin, simplified versions of the non-fading memory constitutive equation must be found. These are required for mathematical convenience and efficiency. Such simplifications might be just the first few terms in a series expansion of the theory or a restricted form of the kernel functions. The forms selected will have to be carefully evaluated to determine if they permit the type of non-linear behavior exhibited by propellants.

Initial tests are underway to answer the following fundamental questions:

a. Do compressive and tensile strain states, of the same absolute magnitudes, result in the same permanent memories (i.e., do they cause the same damage)?

b. Will a tensile strain in a given direction cause a "stress softening" effect for a subsequent tensile strain in a perpendicular direction (i.e., does the material remain isotropic or are there "strain induced anisotropy" effects)?

c. Can the simplified forms of Farris' constitutive equation, that are determined by examining uniaxial test data, be used to predict the results of a biaxial test?

2. Analytical Methods

To check the validity of the new constitutive equation for propellants, it will be necessary to have an efficient procedure for structurally analyzing at least simple models with a mathematical formulation which contains the new constitutive law. Due to the complexity involved this will require a numerical solution which can be evaluated by a computer.

The linear viscoelastic formulation[17] currently used in the transient thermal analysis of cylinders will be utilized as the basis for the analytical development. It is anticipated that most of the effort previously expended in formulating and coding the linear viscoelastic analysis will be directly applicable to the non-linear characterization. In particular, the geometry, heat transfer, and time marching computational aspects of the existing program should be directly usable in the new approach.

As a minimum accomplishment on this program a workable computer program to analyze cylindrical cross-sections of case bonded propellant grains, with non-linear material properties, will be written and checked out.

3. Model Tests

To determine if a constitutive law is predicting properly one must have the ability to analyze and measure the stresses and deformations in certain model structures. This is especially true with thermoviscoelastic analyses where simultaneously we have both thermal and strain gradients throughout the body. Although it is rather easy to model small motors that will produce the same final state of stress and strain as that in a full-scale motor, it is impossible to adjust the time scales to account fully for thermal disturbances. For this reason, and because of the difficulty in measuring stress and strain at all points in a motor, our initial testing is designed to minimize thermal gradients by permitting only very low rates of heating. Tests allowing thermal gradients will be withheld until our analytical relations have been verified under the simpler conditions.

## III. CUMULATIVE DAMAGE ANALYSIS WITH STRAIN FAILURE CRITERIA

Strain failure criteria have been widely used in the solid propellant industry, the success depending on how well the stress-strain distribution and rate of loading conditions of the motor were simulated in the specimens used to measure the properties. Thus multiaxial tests have become standard for full evaluation of failure behavior, even though the uniaxial test remains the basic test for quality control, aging evaluation, and other applications where relative quality is the chief need.

A. BACKGROUND

The use of strain failure data in estimating life began initially with the development of the constant strain test by Aerojet, and the results proved valuable in the development of Polaris and other solid propellants which demonstrated good field performance. The extension to prediction of life from constant rate tests was shown for a tactical solid propellant[19] and subsequent field results have shown good agreement with predicted failure rates. In examining the methods used there, it became clear that strain failure data should be capable of predicting life under varying temperature conditions in a similar manner to the stress criteria methods of Bills.[5-9] This becomes clear if we consider briefly the cumulative damage equations in strain criterion form.

The basic starting relationship[20] is the same

$$\sum D = \frac{1}{P(n)} \sum_{i=1}^{N} \frac{\Delta t_i}{\bar{t}_{fi}} \tag{11}$$

where $\sum D$ is the cumulative damage

$P(n)$ is a statistical distribution parameter and relates the $n^{th}$ test specimen in the distribution to the mean of the population

$\Delta t_i$ is the increment of time the specimen is exposed to the $i^{th}$ "true" stress level

$\bar{t}_{fi}$ is the mean time-to-failure for the population of specimens if the specimens saw only the $i^{th}$ "true" stress level

It can be seen that P(n) can be easily obtained from acceptance tensile data, and the data available indicate the relative quality of a batch in the total population remains essentially constant through life of the propellant.

Now for a strain criterion, we obtain empirically

$$\bar{t}_{fi} = f(T, \varepsilon^*, RH, \varepsilon_{t,T,RH}) \tag{12}$$

where T = temperature seen by the propellant failure region

$\varepsilon^*$ = strain seen by the propellant under the appropriate multiaxial environment

RH = relative humidity seen by the propellant in the actual use condition

$\varepsilon_{t,T,RH}$ = previous strain and environmental history of the propellant.

Equation (11) contains all factors presently felt significant, and which would have to be reproduced for prediction, though experiments would show which were significant for a given propellant in a given environment. The difficulties of controlling conditions under varying strain and temperature in the laboratory are well recognized, so initial experiments would limit the variables to allow inexpensive confirmation of the method.

The solution of Equations (11) and (12) is straightforward for the case of temperature cycling at a low enough rate that temperature equilibrium is attained. This condition lends itself to experimental confirmation using available oven equipment which can change the oven temperature continuously according to a programmed cam. Under the bone dry

conditions of a continuous $N_2$ purge, the control of humidity is simplified and allows experimental measurements of bore strain as a function of time and temperature, together with observation of time to failure.

The data for prediction of the cycling behavior is obtained from tests on samples of the propellant under stress-strain environments closely simulating the motor; the motor itself held at constant temperature to failure thus becomes one type of specimen. A closely related specimen that has been well characterized for stress distribution is the biaxial specimen.[21] Such specimens can be held at constant strain and temperature until failure occurs, or can be pulled at constant rate to failure. Use of constant rate data requires a correction for the time the specimen is at any strain level as it is pulled to failure. Here, too, we can use the correction factor A, developed by Bills[6] to relate the time-to-break, $t_b$, in constant rate testing to the desired value, $t_{fi}$.

$$t_f = t_b A \tag{13}$$

where the value of A is given by

$$A = \int_o^1 \frac{(\varepsilon_t - \varepsilon_{cr})^B}{(\varepsilon_b - \varepsilon_{cr})^B} d(t/t_b) \tag{14}$$

But, since the strain is linear with time in a constant rate test, then for the case of $\varepsilon_{cr} = 0$, $\varepsilon_t/\varepsilon_b = t/t_b$, and $A = 1/(B + 1)$. The resulting data are used to construct graphs as shown in Figure 7 for typical propellant $\varepsilon_b$ vs log $(t_b/a_T)$, using the values in Figure 8 for log $a_T$ vs T. Also, as in Figure 9 , we require predicted or observed $\varepsilon$ vs T from observed strain in the cycled motors. The data can then be read from the curves and tabulated as shown in Table 1 to give the value of $\overline{t}_{fi}$. The values of $\overline{t}_{fi}$ can be used directly to show expected life at any constant temperature

TIME TO FAILURE OF A POLYURETHANE PROPELLANT

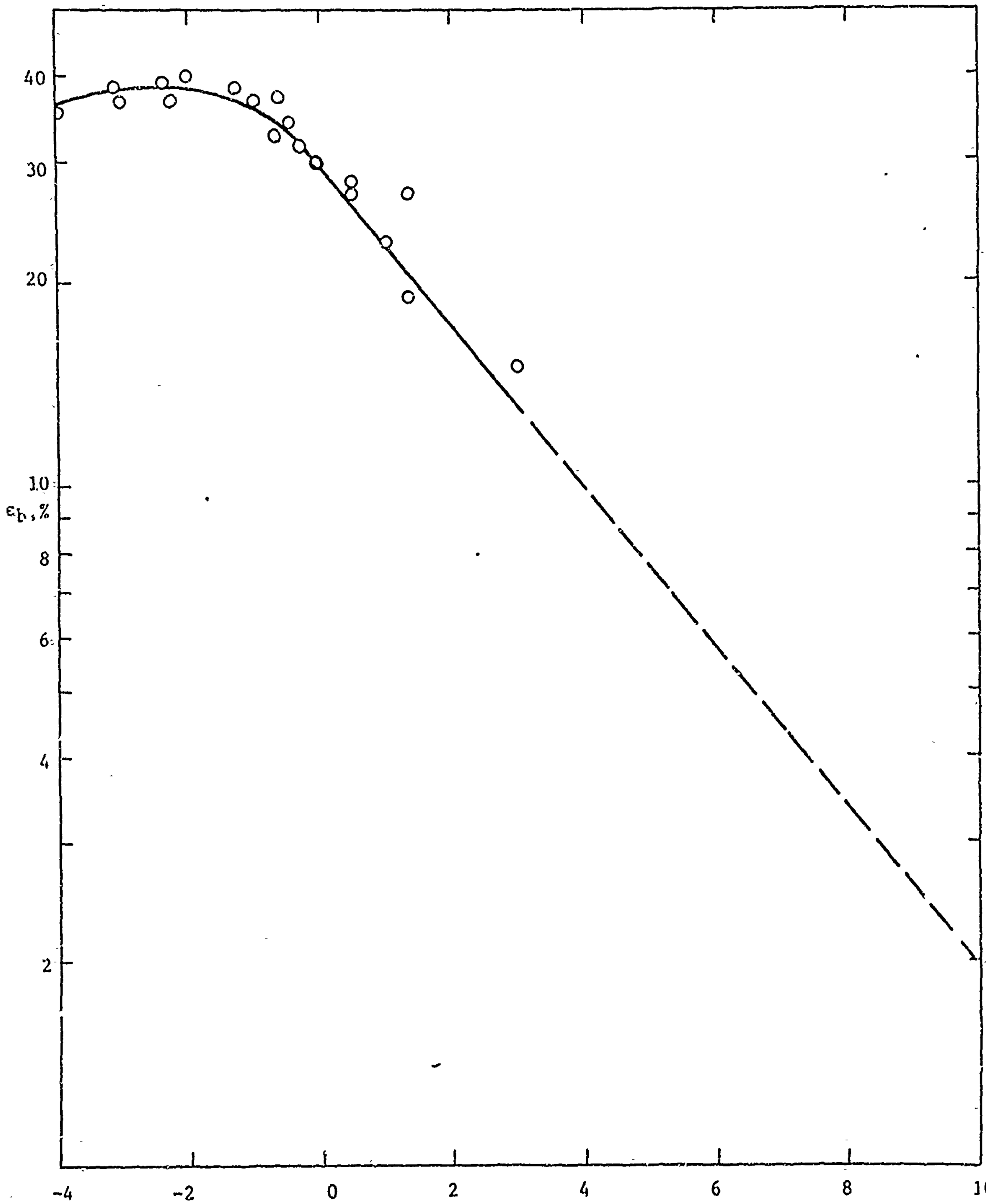



Figure 7

EFFECT OF TEMPERATURE UPON THE VISCOELASTIC SHIFT FACTOR $a_T$ OF A POLYURETHANE PROPELLANT

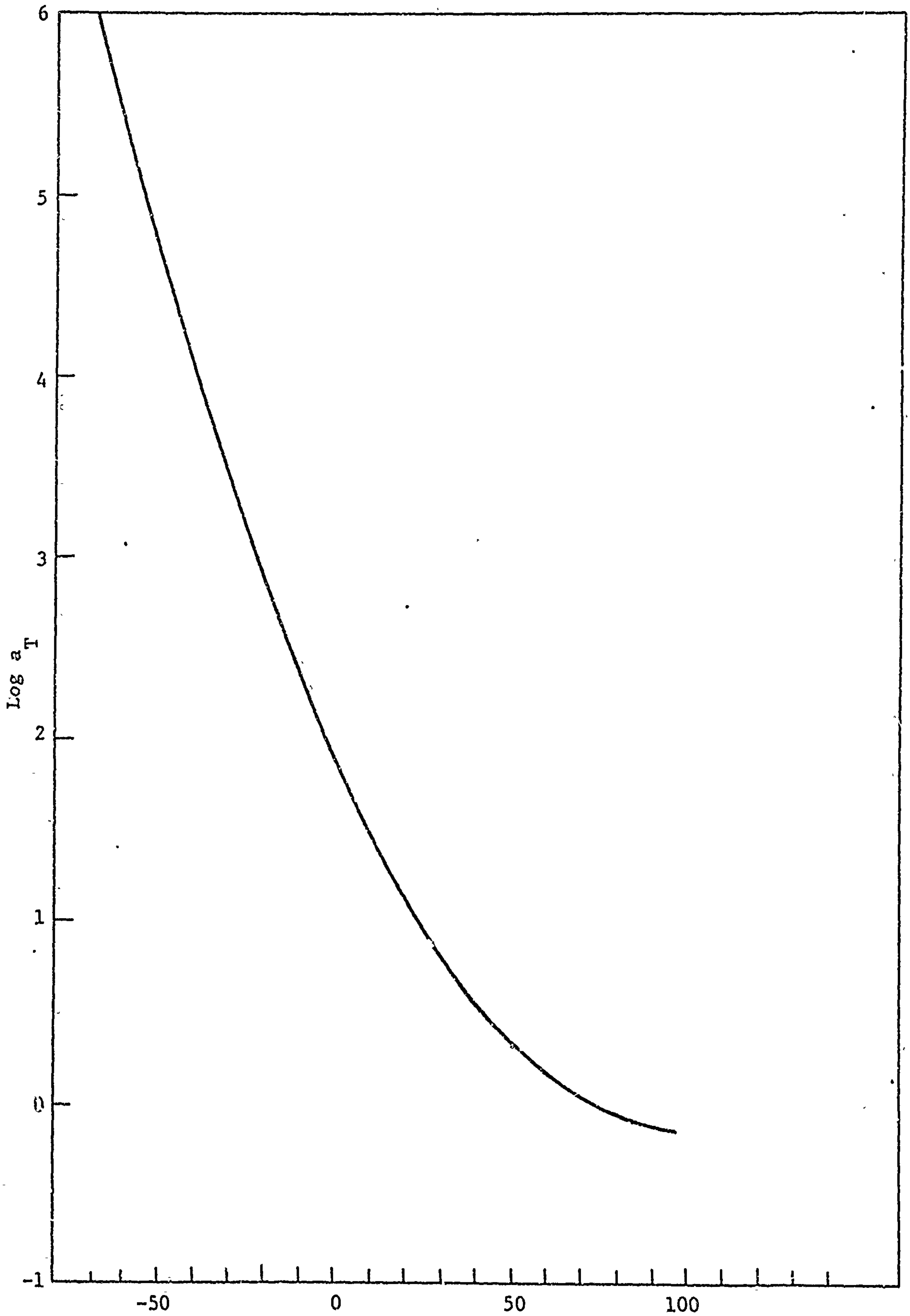



Figure 8

INNERBORE HOOP STRAIN vs. TEMPERATURE FOR TEST MOTOR

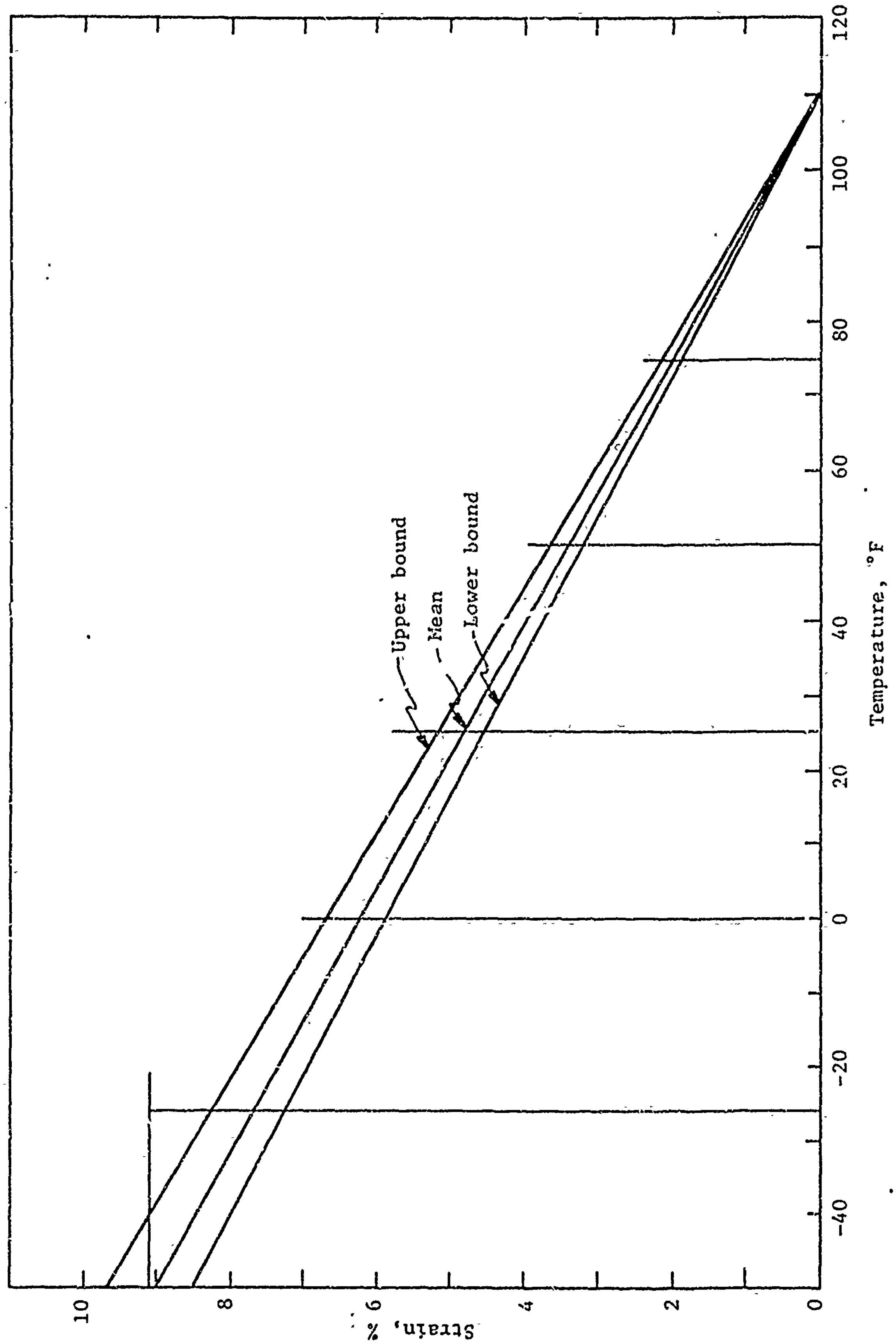





Figure 9

of storage as shown in Figure 10, prepared from the data of Table 1. Figure 10 shows the predicted curve for this same batch of propellant as calculated from cumulative damage analysis; the agreement is surprisingly good in view of the extrapolation required in both predictions.

TABLE 1

SUMMARY OF CALCULATIONS OF STORAGE LIFE
FOR A STRAIN CRITERION OF FAILURE

| Temp., °F | Average Motor Strain,% (From Fig. 9) | $\log(t_f/a_T*)$ | $\log a_T$ (from Fig. 8) | $\log t_f$ |
|---|---|---|---|---|
| 75 | 2.0 | 10.8 | 0.0 | 10.8 |
| 50 | 3.4 | 8.9 | 0.3 | 9.2 |
| 25 | 4.8 | 7.6 | 0.8 | 8.4 |
| 0 | 6.2 | 6.6 | 1.8 | 8.4 |
| -25 | 7.8 | 5.8 | 3.2 | 9.0 |
| -50 | 9.2 | 5.2 | 4.6 | 9.8 |

* Value from Fig. 7, corrected for Factor A

B. CURRENT EFFORTS

A test of applicability of the strain-criterion method of cumulative damage analysis can be carried out simply for the case of thermally cycled, simple tubular grains. It is planned that graphs of strain vs time to failure be prepared from two types of data, first, time to failure of biaxial specimens held at constant strain and time to failure when pulled at constant rates, and second, time to failure of small tubular grains held at constant temperatures; the temperatures selected from the data on the biaxial specimens to give strains causing failures in the desired time ranges. The

FAILURE PREDICTION OF ONE BATCH OF POLYURETHANE PROPELLANT BY STRESS AND BY STRAIN CRITERIA, USING CUMULATIVE DAMAGE METHOD

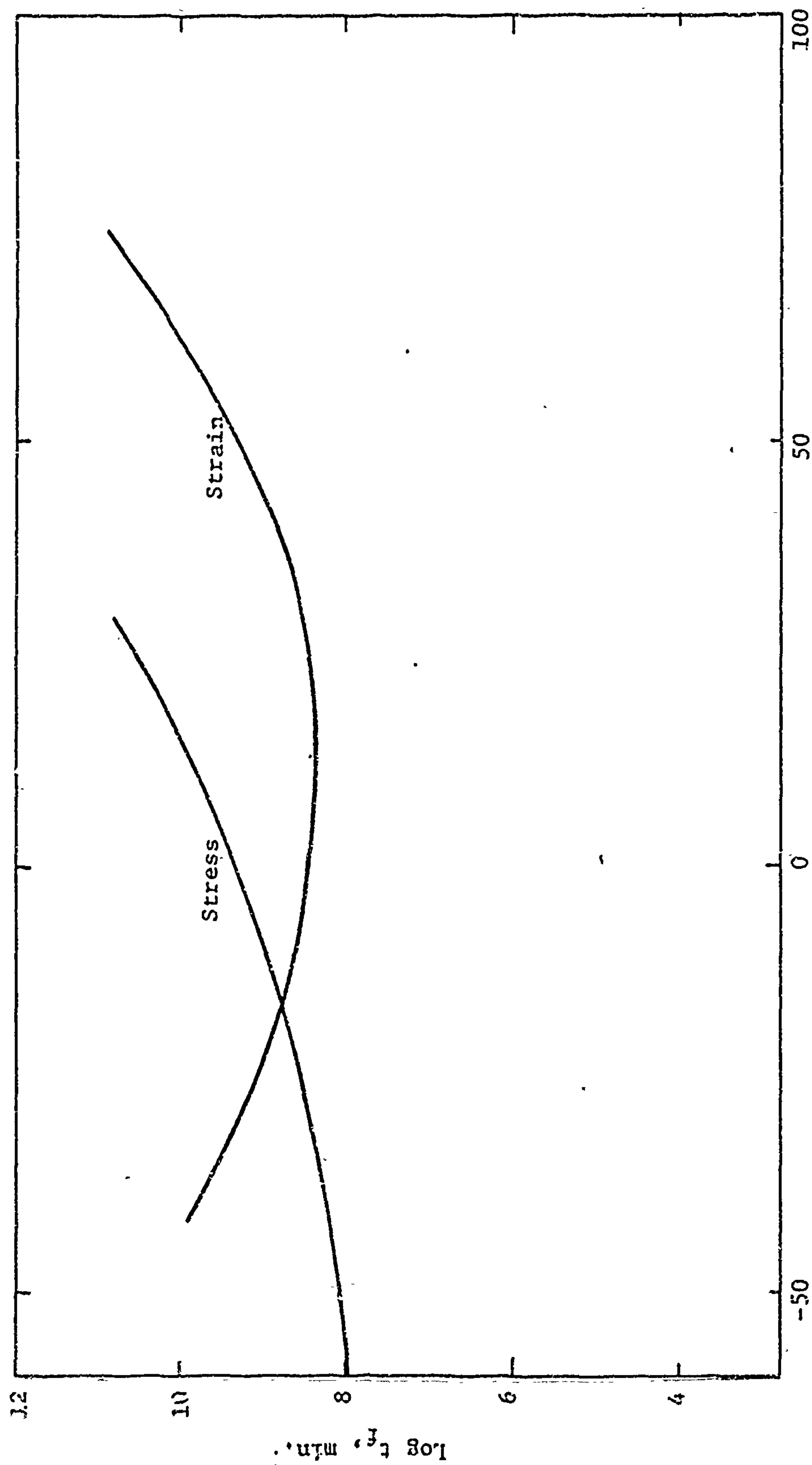



Figure 10

data will be generalized using the $a_T$ values measured for the batch of solid propellant studied, the batch being the same used in the non-linear, viscoelastic characterization studies.

Following the definition of the failure and the $a_T$ graphs, analyses of failure thermally cycled small motors will be attempted.

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13. ABSTRACT

With the development of new propellant materials, like the hydroxyl-terminated polybutadiene propellants (HTPB), grain designs with inner-bore hoop strains in excess of 15% can be employed; while successful performances at strains up to 40% have been observed. At these strain levels the non-linear response effects of propellants are highly significant, even exceeding the broad statistical limits bounding the linear engineering theories. In anticipation of the requirements to be imposed by these newly developing propellants, we plan to evaluate their non-linear viscoelastic behaviors and reduce to practice the associated analytical methods. These studies will center upon the theories recently developed by Farris at the University of Utah. The most advanced and most pertinent failure criterion for solid propellants is the linear cumulative damage criterion. This relation requires, however, a precise definition of the pertinent grain stresses as a function of time. For grains seeing only small strains and simple histories, the existing linear viscoelastic stress analysis is adequate. But, for the larger inner-bore strains, or for more complex histories under small strains, non-linear viscoelastic analyses will probably be required for accurate failure predictions. Pending the development of these analyses an empirical approach to failure predictions is recommended. This approach involves a strain failure criterion in which appropriately designed test specimens are subjected to the same strain-time-temperature history that the grain is expected to experience. A small effort demonstrating this approach is planned.

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| Solid Propellant | | | | | | |
| Mechanical Behavior | | | | | | |
| Failure Criteria | | | | | | |
| Strain Criterion of Failure | | | | | | |
| History Dependence | | | | | | |
| Mechanical Characterization | | | | | | |

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